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10MAT31

Third Semester B.E. Degree Examination, June/July 2014
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Find Fourier series of $f(x) = 2\pi x - x^2$ in $[0, 2\pi]$. Hence deduce $\sum_{1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$. Sketch the graph of f(x). **(07 Marks)**

b. Find Fourier cosine series of $f(x) = \sin\left(\frac{m\pi}{\ell}\right)x$, where m is positive integer. **(06 Marks)**

c. Following table gives current (A) over period (T):

A (amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98
t (sec)	0	T/6	T/3	T/2	2T/3	5T/6	T

Find amplitude of first harmonic. **(07 Marks)**

2 a. Find Fourier transformation of $e^{-a^2x^2}$ $(-\infty < x < \infty)$ hence show that $e^{-x^2/2}$ is self reciprocal. **(07 Marks)**

b. Find Fourier cosine and sine transformation of

$$f(x) = \begin{cases} x & 0 < x < a \\ 0 & x \geq a \end{cases}$$ **(06 Marks)**

c. Solve integral equation $\int_0^{\infty} f(x)\cos sx\,dx = \begin{cases} 1-s & 0 < s < 1 \\ 0 & s \geq 1 \end{cases}$. Hence deduce $\int_0^{\infty} \frac{1-\cos x}{x^2}dx = \frac{\pi}{2}$. **(07 Marks)**

3 a. Find various possible solution of one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by separable variable method. **(07 Marks)**

b. Obtain solution of heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial t^2}$ subject to condition u(0, t) = 0, u(ℓ, t) = 0, u(x, 0) = f(x). **(06 Marks)**

c. Solve Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to condition u(0, y) = u(ℓ, y) = u(x, 0) = 0; $u(x, a) = \sin\left(\frac{\pi x}{\ell}\right)$. **(07 Marks)**

4 a. The revolution (r) and time (t) are related by quadratic polynomial $r = at^2 + bt + c$. Estimate number revolution for time 3.5 units, given

Revolution	5	10	15	20	25	30	35
Time	1.2	1.6	1.9	2.1	2.4	2.6	3

(07 Marks)

b. Solve by graphical method,
Minimize $Z = 20x_1 + 10x_2$ under the constraints $2x_1 + x_2 \geq 0$; $x_1 + 2x_2 \leq 40$; $3x_1 + x_2 \geq 0$; $4x_1 + 3x_2 \geq 60$; $x_1, x_2 \geq 0$. **(06 Marks)**

c. A company produces 3 items A, B, C. Each unit of A requires 8 minutes, 4 minutes and 2 minutes of producing time on machine M_1, M_2 and M_3 respectively. Similarly B requires 2, 3, 0 and C requires 3, 0, 1 minutes of machine M_1, M_2 and M_3. Profit per unit of A, B and C are Rs.20, Rs.6 and Rs.8 respectively. For maximum profit, how many number of products A, B and C are to be produced? Find maximum profit. Given machine M_1, M_2, M_3 are available for 250, 100 and 60 minutes per day. **(07 Marks)**

PART – B

5 a. By relaxation method, solve $-x+6y+27z=85$, $54x+y+z=110$, $2x+15y+6z=72$. **(07 Marks)**

b. Using Newton Raphson method derive the iteration formula to find the value of reciprocal of positive number. Hence use to find $\frac{1}{e}$ upto 4 decimals. **(06 Marks)**

c. Using power rayley method find numerical largest eigen value and corresponding eigen vector for $\begin{bmatrix} 10 & 2 & 1 \\ 2 & 10 & 1 \\ 2 & 1 & 10 \end{bmatrix}$ using $(1, 1, 0)^T$ as initial vector. Carry out 10 iterations. **(07 Marks)**

6 a. Fit interpolating polynomial for f(x) using divided difference formula and hence evaluate f(z), given f(0) = - 5, f(1) = -14, f(4) = -125, f(8) = -21, f(10) = 355. **(07 Marks)**

b. Estimate t when f(t) = 85, using inverse interpolation formula given : **(06 Marks)**

t	2	5	8	14
f(t)	94.8	87.9	81.3	68.7

c. A solid of revolution is formed by rotating about x-axis, the area between x-axis, lines $x = 0$, $x = 1$ and curve through the points with the following co-ordinates.

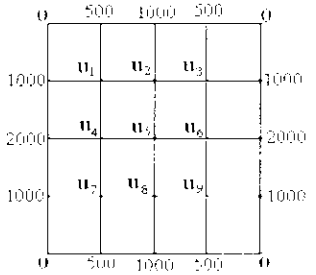
x	0	1/6	2/6	3/6	4/6	5/6	1
y	0.1	0.8982	0.9018	0.9589	0.9432	0.9001	0.8415

by Simpson's $3/8^{th}$ rule, find volume of solid formed. **(07 Marks)**

7 a. Using the Schmidt two-level point formula solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ under the conditions $u(0, t) = u(1, t) = 0$; $t \geq 0$; $u(1, 0) = \sin \pi x$ $0 < x < 1$, take $h = \frac{1}{4}$ $\alpha = \frac{1}{6}$. Carry out 3 steps in time level. **(07 Marks)**

b. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$ subject to $u(0, t) = u(4, t) = u_t(x, 0) = 0$, $u(x, 0) = x(4-x)$ take $h = 1$ $k = 0.5$. **(06 Marks)**

c. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the square mesh. Carry out 2 iterations. **(07 Marks)**



8 a. State and prove recurrence relation of f-transformation hence find $Z_T(n)$, $Z_T(n^2)$. **(07 Marks)**

b. Find $Z_T[e^{n\theta}\cosh n\theta - \sin(nA+\theta) + n]$. **(06 Marks)**

c. Solve difference equation $u_{n+2} + 6u_{n+1} + 9u_n = n2^n$ given $u_0 = u_1 = 0$. **(07 Marks)**

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10ME32A/AU32A/TL32/MT32

Third Semester B.E. Degree Examination, June/July 2014
Material Science and Metallurgy

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define the following lattice:
   i) Unit cell.
   ii) Space lattice.
   iii) Atomic packing factor.
   iv) Coordination number. **(04 Marks)**

b. Calculate the volume of the zinc crystal structure unit cell by using the following data: Pure zinc has the HCP crystal structure with lattice constants a = 0.2665nm and c = 0.4947nm. **(04 Marks)**

c. State and explain Fick's first law of diffusion. **(04 Marks)**

d. What do you mean by crystalline imperfection? Explain briefly point and scalar/line defects. **(08 Marks)**

2 a. Compare the engineering stress and strain with the true stress and strain for the tensile test of a low carbon steel that has the following test values: **(04 Marks)**

Load applied to specimen	= 75kN
Initial diameter of specimen	= 12.5mm
Diameter of specimen under 75kN load	= 12mm

b. Derive the expression for critically resolved shear stress (CRSS). **(04 Marks)**

c. Describe the dislocation mechanism that enables a metal to be plastically deformed without fracture. **(06 Marks)**

d. What do you mean by linear and nonlinear elastic properties of a material? Explain any two properties briefly. **(06 Marks)**

3 a. Define creep and explain a typical creep curve. **(06 Marks)**

b. What is fracture? State the differences between ductile and brittle fractures. **(08 Marks)**

c. State the factors that affect the fatigue strength of a metal. Explain them briefly. **(06 Marks)**

4 a. State and explain Gibbs phase rule. **(04 Marks)**

b. Explain homogeneous nucleation. Discuss the significance of critical radius of nuclei. **(08 Marks)**

c. What is a solid solution? With suitable examples, explain the different types of solid solutions. **(08 Marks)**

PART – B

5 a. Define:
   i) A phase in a material;
   ii) A phase diagram. **(02 Marks)**

b. Derive the lever rule for the binary phase diagram of two metals A and B completely soluble in each other. **(04 Marks)**

c. Write equations for the following invariant reactions: entectic, peritectic, monotectic, entectoid and peritectoid. How many degrees of freedom exist at invariant reaction points in binary phase diagram? **(06 Marks)**

d. Draw neatly Iron-carbon phase diagram and mark on it all salient temperatures, compositions, and phases. **(08 Marks)**

6 a. Explain the steps to construct TTT diagram. Draw a labeled sketch of TTT diagram for an eutectoid steel. **(10 Marks)**

b. With neat diagrams, explain briefly austempering and martempering heat treatment process. **(10 Marks)**

7 a. Discuss briefly about effects of alloying elements on properties of steel. **(10 Marks)**

b. Explain with phase diagram modification of Al-Si alloys. **(05 Marks)**

c. Write a note on Spheroidal Graphite (SG) iron. **(05 Marks)**

8 a. Explain the following for production of FRP:
   i) Spray lay-up process.
   ii) Pultrusion process. **(10 Marks)**

b. Explain with a neat sketch production of MMC by using powder metallurgy process. **(10 Marks)**

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10ME/AU32B

Third Semester B.E. Degree Examination, June/July 2014
Mechanical Measurements and Metrology

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Sketch and explain the following:
   i) Impherial standard.
   ii) International prototype. **(08 Marks)**

b. Discuss the following standards of measurements with their characteristics:
   i) Line standard.
   ii) END standard. **(08 Marks)**

c. What are the major requirements of slip gauges? **(04 Marks)**

2 a. Define the following with sketch:
   i) Nominal size.
   ii) Basic size.
   iii) Actual size.
   iv) Zero line.
   v) Allowance.
   vi) Fit. **(09 Marks)**

b. Differentiate between hole basis and shaft basis system with sketches. **(07 Marks)**

c. How the plain gauges are classified? **(04 Marks)**

3 a. List the characteristics of a comparator. **(06 Marks)**

b. Explain with sketch the principle of a sine bar. **(06 Marks)**

c. Give the combination of angle gauges to obtain the following angles also sketch the arrangement of gauges: i) 37° 9′ 18″ ; ii) 33° 16′ 42″. **(08 Marks)**

4 a. Describe with sketch 3-wire method of measuring effective diameter of the thread. **(10 Marks)**

b. Explain with sketch measurement of tooth thickness of a spur gear using gear tooth Vernier Caliper. **(10 Marks)**

PART – B

5 a. What is measurement? Explain the fundamental methods of measurement. **(06 Marks)**

b. Explain:
   i) Repeatability.
   ii) Sensitivity.
   iii) Hysteresis. **(06 Marks)**

c. With suitable example, explain the stages of generalized measurement system **(08 Marks)**

6 a. Explain with block diagram working of a general purpose oscilloscope. **(10 Marks)**

b. Explain with sketch:

i) Stylus type oscillograph.

ii) Light beam oscillograph. **(10 Marks)**

7 a. Describe with a neat sketch the working and applications of a proving ring. **(06 Marks)**

b. With a neat sketch, explain the working of a hydraulic dynamometer for the measurement of torque. **(07 Marks)**

c. Explain the Bridgemen gauge with a neat sketch. **(07 Marks)**

8 a. Write a note on thermocouple materials. **(03 Marks)**

b. With a neat sketch, explain the working of a optical pyrometer. **(08 Marks)**

c. Show with sketch how the strain gauges are mounted to measure in the following cases:

i) Axial strain only.

ii) Bending strain only.

iii) Torsion strain only. **(09 Marks)**

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10ME/AU/TL33

Third Semester B.E. Degree Examination, June/July 2014
Basic Thermodynamics

Time: 3 hrs. Max. Marks:100

Note: *1. Answer FIVE full questions, selecting at least TWO questions from each part.*
2. Use of thermodynamic data handbook and steam tables is permitted.

PART – A

1 a. Define the following :
(i) Open system (ii) Closed system and (iii) Isolated system , and classify the following into open closed and isolated system and explain giving reasons:
(i) Radiator of a car (ii) Thermos flask (iii) Water pump & (iv) Pressure cooker. **(07 Marks)**

b. What are international fixed reference points? Name few of them. What is their importance? **(05 Marks)**

c. Define a new temperature scale 'N' in which freezing point and boiling point of water are 100 °N and 300 °N respectively. Correlate this temperature scale with centigrade scale for which freezing and boiling points are 0°C and 100°C respectively. **(08 Marks)**

2 a. Starting from a convenient common state point, on P-V diagram, show the four expansion processes for $n = 0$, $n = 1$, $n = \gamma$ (where γ is specific heat ratio) and $n = \infty$, what are each processes called? Indicate their names adjacent to the processes on the diagram. **(06 Marks)**

b. Show that work and heat are path functions. **(04 Marks)**

c. To a closed system 150 kJ of work is done on it. If the initial volume is 0.6 m^3 and pressure of system varies as follows:

$$P = (8 - 4V),$$

where 'P' is pressure in bar and 'V' is volume in m^3. Determine the final volume and pressure of the system. **(10 Marks)**

3 a. Write the steady flow energy equation for an open system and explain the terms involved in it, and simplify SFEE for the following systems:
(i) Steam turbine and (ii) Nozzle. **(06 Marks)**

b. The properties of a certain fluid are related as follows:

$$u = 0.718t + 196$$
$$Pv = 0.287(t + 273),$$

where 'u' is specific internal energy (kJ/kg), 't' is temp in °C, 'P' is pressure in (kN/m^2) and 'v' is specific volume in (m^3/kg). For this fluid find c_p and c_V (ie., specific heat at constant pressure and specific heat at constant volume). If a system composed of 2 kg of this fluid expands in a frictionless piston and cylinder machine from an initial state of 1 MPa, 100°C to a final temperature of 30°C. If there is no heat transfer, find the net work for the process. **(08 Marks)**

c. A blower handles 1 kg/s of air at temperature of 20°C and consumes a power of 15 kW. The inlet and outlet velocities of air are 100 m/sec and 150 m/sec respectively. Find the exit temperature of air, assuming adiabatic conditions. Take $c_p = 1.005$ kJ/kg K. **(06 Marks)**

4 a. State and prove Carnot's theorem. **(10 Marks)**

b. A heat engine is used to drive a heat pump. The heat transfer from the engine and heat pump are used to heat water circulating through the radiators of a building. The efficiency of the heat engine is 27 percent and coefficient of performance of heat pump is 4. Evaluate the ratio of the heat transfer to the radiator circulating water to the heat transfer to the engine. **(10 Marks)**

PART – B

5 a. State and prove Clausius inequality? What is the significance of Clausius inequality? **(10 Marks)**

b. An adiabatic vessel contains 85 kg of oil at a temperature of 27°C. A spherical ball made of steel of 10 kg at 727°C is immersed in oil. Determine change in entropy for the universe. Take specific heat of oil = 3.5 kJ/kg K ; Specific heat of steel ball = 0.5 kJ/kg K. **(10 Marks)**

6 a. Define dryness fraction of steam? What are methods used to measure dryness fraction? With neat sketch explain any one method. **(10 Marks)**

b. A rigid vessel of 2 m^3 volume is filled with superheated steam at 20 Bar and 350°C. The vessel is cooled until the steam is just dry saturated. Calculate the mass of steam in the vessel; the final pressure of steam and amount of energy transferred as heat to the surroundings. **(10 Marks)**

7 a. Write Maxwell relations and explain the terms involved. **(04 Marks)**

b. Show that the change in entropy when a perfect gas undergoes a polytropic change PV^n = Constant is given by the expression

$$(S_2 - S_1) = C_V\left(\frac{r-n}{n}\right) \ell n \left(\frac{P_1}{P_2}\right)$$ **(06 Marks)**

c. 0.2 kg of air with pressure 1.5 bar and temperature 27°C is compressed to a pressure of 15 bar according to the law $PV^{1.25}$ = constant. Determine (i) work done on or by the air (ii) Heat flow to or from the air (iii) Change of entropy stating whether it is an increase or decrease in entropy. For air R = 0.287 kJ/kg K, r = 1.4, C_V = 0.718 kJ/kg K. **(10 Marks)**

8 a. Write notes on the following :
(i) Dalton's law of partial pressures
(ii) Vander Waal's equation of states
(iii) Generalized compressibility chart **(12 Marks)**

b. Determine the pressure exerted by carbon dioxide in a container of 1.5 m^3 capacity when it contains 5 kg at 27°C by using
(i) Ideal gas equation (ii) Vander Waal's equation
Take the Vander Waal's constant a = 365.6 kN-m^4 / $(kgmol)^2$; b = 0.0428 m^3/kgmol
Universal gas const = $\overline{R}$ = 8.3144 kJ/kgmol K. **(08 Marks)**

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10ME/AU/PM/TL34

Third Semester B.E. Degree Examination, June/July 2014
Mechanics of Materials

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions, selecting at least TWO questions from each part.***
2. Missing data may be suitably assumed.

PART – A

1 a. Define (i) Stress (ii) Hook's law (iii) Elasticity (iv) Lateral strain. **(04 Marks)**

b. Explain stress-strain relationship showing salient points on the diagram. **(06 Marks)**

c. A stepped bar is subjected to an external loading as shown in Fig.Q1(c). Calculate the change in the length of bar. Take E = 200 GPa for steel, E = 70 GPa for aluminium and E = 100 GPa for copper. **(10 Marks)**

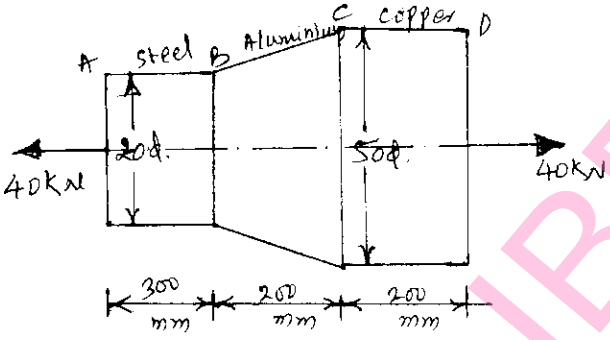

Fig.Q1(c)

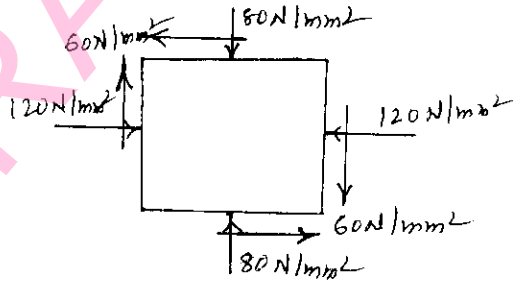

Fig.Q3(c)

2 a. Define (i) Poisson's ratio (ii) Bulk modulus. **(02 Marks)**

b. Derive an expression for establishing the relationship between Young's modulus and modulus of rigidity. **(06 Marks)**

c. A 25 mm diameter steel rod passes concentrically through a bronze tube 400 mm long, 50 mm external diameter and 40 mm internal diameter. The end of the steel rod are threaded and provided with nuts and washers which are adjusted initially so that there is no end play at 20°C. Assuming that there is no change in the thickness of the washers, find the stress produced in the steel and bronze when one of the nuts is tightened by giving it one-tenth of a turn, the pitch of the thread being 2.5 mm. Take E for steel = 200 kN/mm^2 and E for bronze = 100 kN/mm^2. **(12 Marks)**

3 a. Define the principal planes and principal stresses. **(04 Marks)**

b. Explain procedure for constructing Mohr's circle, for an element acted upon by two tensile stresses and shear stresses. **(06 Marks)**

c. The state of stress in two dimensionally stressed body is as shown in Fig.Q3(c). Determine the principal planes, principal stresses, maximum shear stress and their planes. **(10 Marks)**

4 a. Define (i) Strain energy (ii) Work. **(03 Marks)**

b. Prove that volumetric strain in thin cylinder is given by $\frac{Pd}{4tE}(5-4\mu)$, with usual notations. **(07 Marks)**

c. A C.I. pipe has 200 mm internal diameter and 50 mm metal thickness and carries water under a pressure of 5 N/mm^2. Calculate the maximum and minimum intensities of circumferential stress and sketch the distribution of circumferential stress and radial pressure across the section. **(10 Marks)**

PART – B

5 a. Derive the relationship between load, shear force and bending moment. **(05 Marks)**

b. Briefly explain the different types of loads. **(03 Marks)**

c. Draw SFD and BMD for the loading pattern on the beam in Fig.Q5(c). Indicate the point of contraflexure. Also locate the maximum BM with its magnitude. **(12 Marks)**

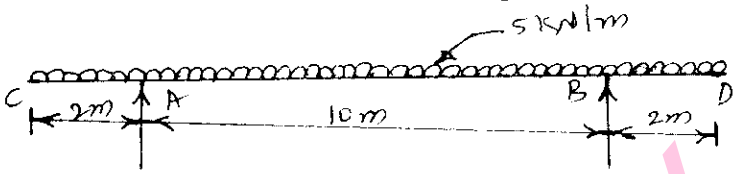

Fig.Q5(c)

6 a. What are the assumptions made in theory of bending? **(04 Marks)**

b. Prove that the maximum shear stress is 1.5 times the average shear stress in a beam of rectangular cross-section. **(06 Marks)**

c. At a given position in a beam of uniform I-section is subjected to a bending moment of 100 kN-m. Plot the variation of bending stress across the section. [Refer Fig.Q6(c)]

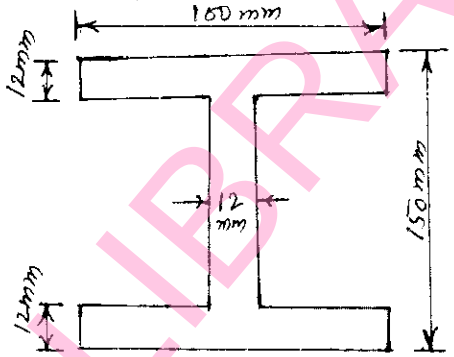

Fig.Q6(c) **(10 Marks)**

7 a. Derive the deflection equation for the beam in the standard form

$$EI\frac{d^2y}{dx^2} = M(x).$$ **(06 Marks)**

b. For the beam loaded as shown in Fig.Q7(b), find the position and magnitude of maximum deflection. Take $I = 4.3\times10^8$ and $E = 200$ kN/mm^2. **(14 Marks)**

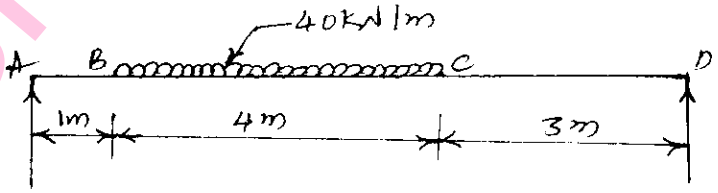

Fig.Q7(b)

8 a. What are the assumptions made in theory of columns? **(03 Marks)**

b. Derive an expression for the critical load in a column subjected to compressive load, when one end is fixed and other end is free. **(07 Marks)**

c. Find the diameter of the shaft required to transmit 60 kW at 150 rpm if the maximum torque is 25% more than the mean torque for a maximum shear stress of 60 MPa. Find also the angle of twist in a length of 4m. Take G = 80 GPa. **(10 Marks)**

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10ME/AU35

Third Semester B.E. Degree Examination, June/July 2014
Manufacturing Process – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. With a neat flow diagram, explain the steps involved in metal casting process. Also write the advantages and limitations and applications of metal casting process. **(12 Marks)**

b. What are pattern allowances? Classify and write a note on draft allowance and distortion allowance with figures. **(08 Marks)**

2 a. Name the base sands used in metal casting and briefly discuss the requirement of base sand. **(10 Marks)**

b. With a neat sketch, explain the working principle of Jolt & Squeeze moulding machine. **(10 Marks)**

3 a. Briefly explain the characteristic features of FURAN and ALKYDE type no bake moulding processes. **(10 Marks)**

b. Name the centrifugal casting methods. With neat sketches explain the working of vertical and horizontal type true centrifugal casting processes. **(10 Marks)**

4 a. With a neat sketch explain the working principle of coreless induction furnace. **(10 Marks)**

b. With a neat sketch explain the working principle of 3-phase electric arc furnace. **(10 Marks)**

PART – B

5 a. With a neat sketch explain the principle of Laser welding process. Mention its advantages, limitations and applications. **(10 Marks)**

b. With a neat sketch explain the thermit welding process. Write the advantages and limitations. **(10 Marks)**

6 a. With a neat sketch explain the working principle of oxy-acetylene gas welding. Also write the flame characteristics. **(12 Marks)**

b. Explain Tungsten inert gas arc welding process with figure. Mention its advantages. **(08 Marks)**

7 Write short notes on the following :

a. HAZ in welding

b. Shrinkage and residual stresses in welding

c. Welding defects and remedies

d. Electrodes **(20 Marks)**

8 a. Distinguish between soldering and brazing. Also discuss briefly the furnace brazing process with figure. **(10 Marks)**

b. With neat sketches, explain the following types of non-destructive methods of inspection of casting and welding:

(i) Eddy current testing (ii) Acoustic emission monitoring. **(10 Marks)**

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MATDIP301

Third Semester B.E. Degree Examination, June/July 2014
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions***

1 a. Find the modulus and amplitude of $\frac{5+3i}{4-2i}$ **(06 Marks)**

b. Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\cos\frac{n\pi}{4}$ **(07 Marks)**

c. Prove that $\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^4 = \cos 8\theta + i\sin 8\theta$ **(07 Marks)**

2 a. Obtain the n^{th} derivative of $e^{ax}\sin(bx + c)$ **(06 Marks)**

b. Find the n^{th} derivative of $\frac{x+3}{(x-1)(x+2)}$ **(07 Marks)**

c. If $y = a\cos(\log x) + b\sin(\log x)$, then prove that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ **(07 Marks)**

3 a. Find the angle of intersection of the curves $r = \sin\theta + \cos\theta$, $r = 2\sin\theta$. **(06 Marks)**

b. Find the pedal equation of the curve $r^n = a^n\cos n\theta$. **(07 Marks)**

c. Using Maclaurin's series expand $\log(1 + \sin x)$ upto the term containing x^4. **(07 Marks)**

4 a. If $z = \frac{x^2+y^2}{x+y}$, then show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$ **(07 Marks)**

b. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **(06 Marks)**

c. If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$, evaluate $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1, -1, 0)$. **(07 Marks)**

5 a. Obtain the reduction formula for $I_n = \int_0^{\frac{\pi}{2}} \sin^n x\, dx$ **(06 Marks)**

b. Evaluate $\int_0^{\pi}\int_{2\sin\theta}^{4\sin\theta} r^3 dr d\theta$ **(07 Marks)**

c. Evaluate $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)dx\,dy\,dz$ **(07 Marks)**

6 a. With usual notations, prove that

$$\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$ **(06 Marks)**

b. Show that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta \times \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$ **(07 Marks)**

c. Prove that $\beta(m, \frac{1}{2}) = 2^{2m-1}\beta(m, m)$ **(07 Marks)**

7 a. Solve $\frac{dy}{dx} = (4x + y + 1)^2$, if $y(0) = 1$. **(06 Marks)**

b. Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ **(07 Marks)**

c. Solve $\left\{y\left(1 + \frac{1}{x}\right) + \cos y\right\}dx + (x + \log x - x\sin y)dy = 0$ **(07 Marks)**

8 a. Solve: $(D^3 + D^2 + 4D + 4)y = 0$ **(06 Marks)**

b. Solve: $(D^2 - 5D + 1)y = 1 + x^2$ **(07 Marks)**

c. Solve: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2x}\sin x$ **(07 Marks)**

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